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# Why Me ?
# Siting a Locally Unwanted Public Good

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# Why Me ?
## Siting a Locally Unwanted Public Good

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**Abstract**

The siting of public facilities, such as prisons, airports or incinerators for hazardous wastes faces social rejection by local population. These public goods have a private bad aspect which creates a siting problem: all communities benefit from its existence, but only one (the host) bears its cost. We tackle this inevitable asymmetry from a responsibility and equity viewpoint: the host should not be perceived as a "victim". To realize this objective, we design a method to share the total cost (the disutility of the host plus the construction cost) in a way that bypasses the natural asymmetry of the problem. We also introduce a basic incentives property which has strangely been overlooked in the existing literature: voluntary participation.

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# 1 Introduction

"You can't park here: it's my retreat[1]". The actor George Clooney has turned civic protester in an attempt to stop the building of parking lots that he says will ruin the peace and beauty of Lake Como (Italy). The "peace and beauty" of his nearby lakeside villa is also mentioned. In a different context, the local residents of the small English village of Ashtead, United Kingdom, claim that they want "No heroes in their backyard"[2]: they are objecting to the conversion of a large residential property into a family support center for relatives of wounded British service personnel. Their main complaint was that increased noise and traffic would "ruin the special character and appeal" of the private lane. Many other similar stories can be found on newspapers around the world, and they share a common property: local opposition to a public project.

The siting of a public facility, such as prisons, airports or incinerators for hazardous wastes face social rejection by local population. These goods are socially necessary but come with local externalities (noise, pollution, noxious odors...) or bear a negative connotation. Different factors can be the cause of such rejection: the loss in the economic value of property, perceived loss in quality of life or the fear of health effects. In economic terms, these public goods have a private bad aspect which creates a siting problem: all communities will benefit from the good, but only one (the host) will bear the cost. This asymmetry typically leads to costly procedures or inefficient siting.

Opposition from the public is identified by numerous studies as a major factor explaining siting difficulties (see Mitchell and Carson (1986), the Environmental Protection Agency (2002), Minehart and Neeman (2002) or Marchetti (2005) for comprehensive reviews). Recently, Vajjhala and Fishbeck (2006) quantify the difficulty to site a locally unwanted public good (U.S Transmission line siting). Combining different datasets based on state-level data from the United States their empirical study shows that public opposition is the main factor explaining siting difficulties (in front of environmental and regulation factors). Solving this siting problem requires that we understand the parameters which generate this self-defense behavior of sorts.

The literature calls this public rejection the *NIMBY* syndrome (Not In My Back-

[1]The Times, June 22, 2007

[2]http://www.yourlocalguardian.co.uk/news/epsomnews/display.var.1558300.0.0.php

Yard). It is the result of a perceived cost for the host community higher than the perceived benefit than it derives from the consumption of the public good. Without compensation it seems difficult for the host to accept the noxious facility. Thus, different compensation schemes have been designed to overcome the issue (Kunreuther and Kleindorfer (1986), Sullivan (1992), O'Sullivan (1993), Minehart and Neeman (2002)...). Most of this literature consists in the design of procedures for choosing one site among a group of alternatives and compensating the host for its local disutility. The main objective is to design a procedure which is *efficient*: construction cost and the loss in welfare to the host community should be minimized. The key problem lies in the management of this second aspect, welfare loss is a subjective matter which brings very little consensus. The traditional approach in the economic literature focuses on the strategic properties of the procedures: They are all mainly designed to make communities reveal their true cost for hosting the public good. Then, the community with the lowest cost is chosen and is compensated in a way which preserves the strategic properties of the method.

We question this exclusivity granted to the strategic properties of the procedures: we take the view that redistribution is intrinsically part of the siting problem. By reviewing four cases of waste disposal facilities in the Canadian context, Khun and Ballard (1998) conclude that inequity perception and political dimension (beyond the economic implications) were the main causes of the NIMBY effect. Similarly, Pol et al. (2006), adopting a social-psychological approach, review previous literature and point out that *"the outcomes and lines of argument [reviewed] present the NIMBY issue in terms of distributive justice, inequity perception and risk attribution."*. Then, they analyze 47 environmental conflicts in Catalonia between 1988 and 2003. They find a perceived inequity in reluctant groups similar to that in Khun and Ballard (1998). They add that *"arguing annoyances, risk, distrust in the technology or its management, and in the decision making of politicians are ways to express this perceived inequity"*.

Thus, inequity perception arises from the physical structure of the problem. Efficiency asks that only one site be chosen for the public good: constructing multiple airports, dumps or prison on a small area is certainly more costly than one for all communities. This inevitable asymmetry generates the perceived inequity ("Why me?"). One cannot change the property of the problem, but one can design a cost-sharing method (assigning to each community a share of the total cost) to overcome rejection by diminishing the inequity perception. We will focus on the notion of *responsibility* to address this issue: communities have to pay, and the host to be compensated, just for the aspects they are responsible for. Despite the inevitable physical asymmetry, our goal is to level the playing field between the participants: all communities have the same responsibilities towards the public good ex-ante (before the host is chosen), the host should not bear more responsibilities ex-post (after it is chosen). We wish ex-post responsibilities to be as close as possible to the ex-ante situation: enforcing ex-post equity, through the equalization of ex-ante and ex-post responsibilities, shall help overcome the siting problem. Naturally, the host bears a strong responsibility: its disutility determines the total cost to be shared among the set of communities. A standard requirement, in more classical cost-sharing literature, is that if the total cost increases, no one should pay less than before; hence in our context, if the disutility of the host is higher, no community should have a lower cost share. Thus, we extend this responsibility to all communities and require that all be subject to that cost monotonicity. We call this property *extended cost monotonicity*.

We take the view that communities are only responsible for their preferences over the public good[3]. The relevant preferences for the problem at hands are twofold: the benefit a community obtains from the consumption of the public good ($b_i$ for a community $i$) and the disutility, $d_i$, a community $i$ endures if it is the host of the public good. We model these preferences as the aggregation a community's inhabitants: thus, its characteristics (or preferences) change as a result of the population movement. In practice, several movements of population are observed once the announcement of the host is made (a point raised by Sullivan (1990) and Baumol and Oates (1998)). Some agents (living in a non-host community) with very low disutilities may move near the public good because of the lower housing prices, or because of other advantages brought by the compensation scheme (improved public infrastructures...) while some agents with high disutility may move away from the host community. This, in turn, may change the communities' characteristics and their

[3]We follow here one of the traditional economic treatment of responsibility and preferences *(Cite Fleurbaey, Moulin...)* and other studies on the NIMBY issue: *"faced with compensatory measures, acceptance and rejection of hazardous facilities will depend on the belief and value system of the affected community" (Pol et al. (06)).*

costs share. To ensure ex-post equity we must address this issue explicitly. To do so, we define the following property: if population movements occur between a subset of communities, and they suffer from it (their aggregate cost shares increases), no community (outside of this subset) should benefit from it. This solidarity property directly translates our argument which states that no community is responsible for the distribution of preferences. If the distribution changes, no community should suffer while others benefit because this change cannot be linked to anyone's specific responsibility. Thus, the host, or any other community, cannot be the only one to unfairly suffer from population movements while others benefit from it. We call this property *Zero Gain Under Endogenous Grouping.*

In addition to these properties, specific to the siting issue, we also require two basics properties: efficiency and voluntary participation. The majority of the studies assessing the NIMBY issue only consider $d_i$, the disutility for hosting the public good. We believe that adding a benefit component enhances the model in at least two ways. First, it determines explicitly wether the public good should be constructed or not (if the sum of the benefits exceed the total cost). Second, and most importantly, it justifies a bound on the cost share each community will be asked to pay. Ignoring the benefits leads to ignoring the voluntary participation of each community, which could be very problematic for the attribution of cost shares (and the siting procedure).

It turns out that only one method meets all the above properties: we call it the *Equal Responsibility Method*, it shares the cost proportionally to the benefits that each community obtains from the consumption of the public good.

# 2 Related Literature

The siting problem has been widely studied in economics by considering the siting of a public bad: each community is identified by a disutility ($d_i$), then an auction-like procedure elicits a site (the community with the lowest disutility, for efficiency) and a compensation scheme is constructed: each community pays an accordingly amount.

The first paper to study the problem of siting a waste treatment facilities in this way is Kunreuther and Kleindorfer (1986). They propose a sealed-bid auction procedure to create an incentive for each community to reveal truthfully their costs (disutility and technical cost for hosting the facility): each community pays its own bid. O'Sullivan (1993), Minehart and Neeman (2002), Perez-Castrillo and Wettstein (2002) also propose auction mechanisms, in the same vein as Kunreuther and Kleindorfer, aiming for efficiency and truthful revelation. The traditional trade off between efficiency, budget balance and strategyproofness is central in these papers.

Taking a different approach, Marchetti and Serra (2004) consider the siting problem as a cooperative game. They study the standard solutions of cooperative game theory (the Shapley value, the nucleolus and the core) with an asymmetry in the value function: the value of the cooperation changes when the host changes. Then, they design an experiment and test which solution is the most appealing for participants.

Sakaï (2006) axiomatizes the properties of the proportional procedure used by Minehart and Neeman (2002). He characterizes this method with the axioms of core property, monotonicity, and reallocation-proofness. The core property is an axiom which implies individual rationality and efficiency, monotonicity states that a community generating more wastes should pay more, and reallocation-proofness states that no coalition of communities should gain by strategically transferring their wastes among the group through side-payments. This property is similar to the No-Merging and Splitting property of the proportional method in the rationing model (see Moulin (2004)). He found the same kind of results than Minehart and Neeman: the procedure is not perfectly strategy-proof, but all deviations are still in the core, making it desirable for each community and coalitions.

In a companion paper, Laurent-Lucchetti and Leroux (2007) design a simple mechanism to choose an efficient site which allows the implementation of any reasonable redistribution scheme. The unique subgame-perfect Nash equilibrium of this mechanism coincides with truthtelling, is efficient, budget-balanced and is immune to coalitional deviations. Thus, it allows to choose a site and to share the cost in a predetermined way so as to achieve normative goals (such as the desirable properties we propose here).

# 3 The model and the desirable properties of the Equal Responsibility Method

Let $N = \{1, ..., n\}$ be the set of communities. Each community $i = 1, ....n$ obtains a benefit, $b_i$, from the consumption of the public good and endures a disutility, $d_i$, if it is the host of the public good. Let $b = (b_i)_{i \epsilon N}$ be the profile of benefits and $d = (d_i)_{i \epsilon N}$ be the disutility profile. We consider that the cost of construction of the public good is a constant added to the disutility of all communities.

The set of communities jointly determines a decision function $a : \mathbb{R}^N_+ \times \mathbb{R}^N_+ \mapsto N \cup \emptyset$ within the finite set $\mathcal{A}$. This decision, determined by the profile $(b, d)$, relates to the construction, or not, of the public good and to the choice of a host ("1" if the host is community 1, "2" if the host is community 2... and $\emptyset$ if not built). The communities share the cost of the public good (if it is built). The total cost to be shared is equal to the disutility of the host of the public good. For efficiency, the community with the lowest disutility should be chosen to be the host. A cost-sharing method assigns a vector of nonnegative cost-shares $x(b, d) \in \mathbb{R}^N_+$ such that $\sum_N x_i(b, d) = min_i(d_i)$.

Without loss of generality we rank communities from lowest to highest disutilities:: $d_1 \leq d_2 \leq .... \leq d_n$. Thus a decision $a$ is *efficient* if: when $\sum_N b_i \geq d_1$, $a(b, d) = 1$, else, $a(b, d) = 0$. We consider from now on only the set of efficient solutions, so community 1 will be referred to as the host of the public good and the total cost to be shared will be $d_1$. Moreover, we only consider the case $a(b, d) = 1$ (the public good is constructed).

To overcome the natural asymmetry of the problem (one community which bears the cost for the benefits of all) we define some properties for the cost sharing method which aim to level the responsibilities among communities. The first set of properties are standard fares in the distributive justice literature. The objective is to insure basic fairness (and incentive-compatibility) of the cost sharing method. Then, we add two specific properties, relevant for the specificity of the siting problem.

The first property is a basic incentives property which has been strangely overlooked in the existing literature: voluntary participation (communities should not pay more than the benefits they obtain from the consumption of the public good). The basic assumption made by the authors of previous papers is that the stand alone of each community is to be the host of a public good. We consider here that the public good brings some benefits to the communities and, consequently, that the stand alone of each communities is to agree to have the public good sited on its territory only if the benefits it obtains is superior to the disutility it endures. Moreover, if we want our solution to be stable (no community loses by participating to the procedure) we have to require that no community pays more than the benefits it obtains from the consumption of the public good.

**Voluntary Participation (VP)**: For all $b, d, i$, $x_i(b,d) \leq b_i$.

The second property translates our statement that communities are responsible for their own preferences. It states that if a community is in a profile in which it obtains higher benefits from the consumption of the public good, all else equal, then it could not obtain a lower cost share.

**Monotonicity in benefits (b-MON)**: For all $b, b'$ and $i$, $b_i \leq b'_i \Rightarrow x_i((b'_i, b_{-i}), d) \geq x_i(b,d)$.

The same argument, valid for the disutility (if a community is in a profile in which it has a higher disutility, all else equal, then it could not obtain a lower cost share), is implied by the following property. This property is more specific to the siting problem. The host naturally bears a strong responsibility: its disutility determines the total cost to be shared among the set of communities. A standard requirement, named cost monotonicity, states that no community should pay less if the total cost (the disutility of the host) is higher. Because the solution should not treat the host asymmetrically, *Extended Cost Monotonicity* extends the responsibility of the host community to the collectivity. It states that each community should be equally responsible for its disutility and all communities are subject to cost monotonicity.

**Extended Cost Monotonicity (ECM)**: For all $d \ll d'$, $i$ and $j$, $x_j(b,(d'_i, d_{-i})) \geq x_j(b,d)$

The last property we introduce imposes a solidarity between communities when population movements occur. It states that if communities suffer (i.e. they obtain a higher total cost share), then no other community should benefit. This property directly translates our argument which states that no community is responsible for the distribution of preferences. If the distribution changes, then no community should suffer from it while others benefit from it, because this change could not be linked to anyone's specific responsibility.

**Zero Gain Under Endogenous Grouping (ZGUEG)**: For all $b$, $b'$, $d$, $d'$, $S$, $j$ and $i$,

$$\left.\begin{array}{c} \sum_{j\epsilon N} d'_j = \sum_{j\epsilon N} d_j \\ \sum_{j\epsilon N} b'_j = \sum_{j\epsilon N} b_j \\ min(d_i) = min(d'_i) \\ \sum_{j\epsilon S} x_j(b', d') \geq \sum_{j\epsilon S} x_j(b, d) \end{array}\right\} \implies x_i(b', d') \geq x_i(b, d), \forall i\epsilon N \backslash S.$$

## 4 The Equal Responsibility Method

Let $b_N = \sum_N b_i$. Then, the *Equal Responsibility Method* is:

$$x_i(b, d) = \frac{b_i}{b_N} d_1 \tag{1}$$

**Theorem**: Given n $\geq$ 3, the *Equal Responsibility Method* is the only method which meets *Voluntary Participation*, *Monotonicity in b*, *Extended Cost Monotonicity* and *Zero Gain Under Endogenous Grouping.*

**Proof**: See Appendix.

An intuitive way to split cost, the constrained egalitarian method (where the total cost is equally split among communities, taking into account the voluntary participation constraint) fails to pass the *ZGUEG* property. Indeed, when population movement occurs, it is possible for certain communities to benefit from the change in distribution while the affected communities will suffer from it: a movement from an unconstrained community which gives a higher benefits to a constrained community, everything else equal, will make their aggregate cost share higher and other will benefit from it. Also, the proportional sharing of the total cost with respect to disutilities (the $d_i$) obviously fails the extended cost monotonicity property: the host still bears a higher responsibility, and may reject the public facility.

The planner interested in the implementation of the Equal Responsibility Method must obtain some information on $b$, the profile of benefits, and $d$, the profile of disutilities. In a companion paper, Laurent-Lucchetti and Leroux (2007) present a mechanism which permits to select an efficient host, while allowing any redistribution schemes to be implemented. This paper also presents a procedure, inspired by the one of Jackson and Moulin (1992), which permits to obtain the profile of benefits as well. This procedure allows the implementation of the Equal Responsibility Method.

# 5 Conclusion

We presented a simple solution for solving the siting problem. The objective was to site the public good and share its cost in a way that minimized the chance of rejection. Our aim was to capture the specificity of the problem (one community which bears the cost, benefits accrue to all) and overcome it by an appropriate method. The traditional economic literature focuses on the strategic properties of the procedure that elicit and compensate the host. By arguing that inequity perception is central in the siting problem, we preferred to focus on the distributive justice properties of the method: the aim was mainly to overcome the natural asymmetry, the host should no longer be perceived as a "victim". Thus, we designed a method which level the playing field among the communities. We defined some natural desirable properties and found that only one cost sharing method meets all of this property: the Equal Responsibility Method. We feel that the uniqueness and the simplicity of the solution are significant advantages for a concrete application.

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# A Proof of Theorem

Let $x$ be a cost sharing method which meets *Voluntary Participation*, *Extended Cost Monotonicity* and *Zero Gain Under Endogenous Grouping.*

***Step 1:*** Take a profile $(b, d)$. Let $d'$ be such that $d'_1 = d'_2 = ... = d'_n = d_1$. Then, by *ECM* no cost share with this profile $(b, d')$ should be higher than with the profile $(b, d)$: $x_i(b, d) \geq x_i(b, d')\ \forall i \in N$. By budget balance the cost shares actually have to be the same in the profile $(b, d)$ and $(b, d')$:

$$x(b, d) = x(b, d')\ for\ all\ b, d\ and\ d'\ s.t.\ d'_1 = d_1. \tag{2}$$

The cost shares of each agent is only determined by the profile $b$ and the total cost $d_1$. From know on we will slightly abuse notation and write $x(b, d_1)$ instead of $x(b, d)$.

***Step 2:*** We now show that, by *ZGUEG* and budget balance, the cost share of an agent $i$ is determined by $b_i$, $\sum_{j \neq i} b_j$ and $d_1$. Consider the case of $n$ communities:

$$x_1(b, d_1) + ... + x_i(b, d_1) + ... + x_n(b, d_1) = d_1 \tag{3}$$

Now, let $b'$ be such that $b'_i = b_i$ and $b'_{N \backslash i} = b_{N \backslash i}$. By *ZGUEG* agent $i$ should not obtain a higher cost share if $\sum_{N \backslash i} x_j(b', d_1) \leq \sum_{N \backslash i} x_j(b, d_1)$ (or a lower cost share if the sum in the "prime" profile is higher than in the original profile). By budget balance $\sum_{N \backslash i} x_j(b', d_1)$ actually must be equal to $\sum_{N \backslash i} x_j(b, d_1)$, which implies $x_i(b', d_1) = x_i(b, d_1)$. So, $x_i(b, d_1)$ depends only upon $b_i, \sum_{j \neq i} b_j$ and $d_1$ for all $i \in N$.

***Step 3:*** Following Step 2, the $n$ communities case can be rewritten:

$$x_1(b_1, b_N, d_1) + ... + x_i(b_i, b_N, d_1) + x_j(b_j, b_N, d_1) + ... + x_n(b_n, b_N, d_1) = d_1 \tag{4}$$

Let $i, j \in N$ and $b'$ be such that $b'_i = b_i + b_j$, $b'_j = 0$ and $b'_k = b_k\ \forall\ k \neq i, j$. We know, by Step 2, that $x_i(b'_i, b_N, d_1) + x_j(b'_j, b_N, d_1) = x_i(b_i, b_N, d_1) + x_j(b_j, b_N, d_1)$. By *VP*, $x_j(b'_j, b'_N, d_1) = 0$. Thus:

$$x_i((b_i + b_j), b_N, d_1) = x_i(b_i, b_N, d_1) + x_j(b_j, b_N, d_1). \tag{5}$$

Given $d_1$ and $b_N$, the cost share of each community $i$ is only determined by its $b_i$. Again, we slightly abuse notations and rewrite equation (5):

$$x_i((b_i + b_j)) = x_i(b_i) + x_j(b_j). \tag{6}$$

Which holds for all $b_i, b_j$ such that $b_i \in (0, b_N)$, $b_j \in (0, b_N)$ and $(b_i + b_j) \in (0, b_N)$.

Now, let $b''$ be such that $b''_j = b_i + b_j$, $b''_i = 0$ and $b'_k = b_k \; \forall \; k \neq i, j$. Then we could apply the same argument and found that:

$$x_j((b_i + b_j)) = x_i(b_i) + x_j(b_j). \tag{7}$$

holds for all $b_i, b_j$ such that $b_i \in (0, b_N)$, $b_j \in (0, b_N)$ and $(b_i + b_j) \in (0, b_N)$. We could conclude that:

$$x_j \equiv x_i \; on \; (0, b_N) \; \forall \; i, j. \tag{8}$$

By combining this result, equation (5) and equation (7) we obtain:

$$x_i((b_i + b_j)) = x_i(b_i) + x_i(b_j). \tag{9}$$

Which is a Cauchy functional equation.

***Step 5:*** We have to prove that the general solution of such a functional equation is a linear function. We will follow the proof presented in Aczél (1966). Define $F : [0, b_N] \to \mathbb{R}_+$. Let $F(z) = x(b_i)$ and $F(y) = x(b_j)$. Then we saw that:

$$F(z + y) = F(z) + F(y) \tag{10}$$

is valid for those $z$, $y$ which satisfy $0 \leq h \leq b_N$, $0 \leq y \leq b_N$ and $0 \leq z + y \leq b_N$. Then, by induction:

$$F(z_1 + ... + z_n) = F(z_1) + ... + F(z_n) \tag{11}$$

By setting $z_1 = z_2 = ... = z_n$ it follows that

$$F(nz) = nF(z) \tag{12}$$

is valid for $nz \in [0, b_N]$. In particular for $z = b_N/n$:

$$F(b_N) = nF(b_N/n) \tag{13}$$

with $F(b_N) = \lambda$

$$F(b_N/n) = \frac{\lambda}{n} \tag{14}$$

which implies, by (12)

$$F(mb_N/n) = mF(b_N/n) = \lambda \left(\frac{m}{n}\right) \tag{15}$$

for $m \leq n$, so

$$F(z) = \lambda h \tag{16}$$

holds for rational points in the interval $(0, b_N]$.

We now have to prove that this is the case for all real $h$ in our domain. By *b-Mon*[4]

$$F(z+y) \geq F(z) \tag{17}$$

Let $\{r_k\}$ be an increasing and $\{R_k\}$ be a decreasing sequence of rational numbers converging towards $z$. Then we have for each $k$:

$$r_k < z < R_k \tag{18}$$

and,

$$F(r_k) \leq F(z) \leq F(R_k) \tag{19}$$

that is,

$$\lambda r_k \leq F(z) \leq \lambda R_k \tag{20}$$

from which it immediately follows

[4]Formally, *b-mon* is unnecessary for the proof, it is implied by *VP* and the additive form of $F(.)$. However, this property is desirable in our context, we use it explicitely.

$$F(z) = \lambda z \tag{21}$$

for all $z$ in our domain, which was to be proved.

***Step 6:*** We have to prove, to conclude the proof, that $\lambda = \frac{d_1}{b_N}$. We know that:

$$x(b_i) = \lambda b_i \tag{22}$$

and,

$$x(b_1) + x(b_2) + ... + x(b_n) = d_1 \tag{23}$$

and also,

$$\lambda b_1 + \lambda b_2 + ... + \lambda b_n = d_1 \tag{24}$$

that is,

$$\lambda = \frac{d_1}{b_N} \tag{25}$$

Which implies,

$$x_i(b, d) = b_i \frac{d_1}{b_N} \tag{26}$$

Which is the *Equal Responsibility Method.*

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